LEE HOCK LEONG
M.Sc. (Pure Maths) – CGPA 4
B.Sc with Ed. (1st Class Hons.)

NEGERI SEMBILAN (2006)
MATHEMATICS T (PAPER 1)

1. Solve the simultaneous equations,

   $x \log_8 32 - y \log_4 2 = \frac{3}{2}$ and $\log_2 x + \frac{1}{2}\log_2 y^2 = 2\log_4 3$. [5]

2. Find the solution set of the inequality $\frac{3}{1-x} \le 5 - 4x$. [5]

3. Express $\frac{x^2 - 3x}{x^2 - 2x + 1}$ in partial fractions. [5]

4. The sum, S_n, of the first n terms of an arithmetic progression is given by

   $S_n = pn + qn^2$. Given also that $S_3 = 6$ and $S_5 = 11$.

   (a) Find the values of p and q, [3]

   (b) Deduce an expression for the n^{th} term and the value of the common difference. [3]

5 (a) Find $\int \frac{x}{(x+1)^2}\,dx$. [4]

   (b) Use the trapezium rule with ordinates $x = 0, 1, 2, 3, 4$ to estimate the value of

   $\int_0^4 \frac{1}{1+\sqrt{x}}\,dx$ giving your answer correct to 3 significant figures . [4]

6. Given that the circle has equation $x^2 + y^2 - 18x - 6y + 45 = 0$.

   (a) Find the radius and the centre C of the circle. [3]

   (b) Find the shortest distance from C to the line $x - y = 0$. [2]

   (c) If N is the foot of perpendicular of C to $x - y = 0$, find the area of triangle OCN, where O is the origin. [3]

7. Functions f and g are defined by, $f : x \to |x-2|, x \in \mathcal{R}$, $g : x \to x^2 - 10, x \in \mathcal{R}$.

   (a) Sketch the graph of f and explain why f^{-1} does not exist. [3]

   (b) Write an expression for $fg(x)$ and sketch the graph of fg.
   Hence solve the equation $fg(x) = x$. [7]

8. Given a polynomial $f(x) = x^3 - 19x + 30$. Show that $(x-2)$ is a factor of $f(x)$.

   Hence, solve the equation $f(x) = 2(x-2)$. [8]

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9. Expand $\frac{1}{(a+bx)^2}$, in ascending powers of x up to and including the term in x^2.

   The coefficients of x and x^2 in the expansion are equal, show that $3b + 2a = 0$.

   Given that the sum of the constant term and the coefficient of x^2 is 84, find the possible values of a and b.

   Hence, state the set of values of x for which the expansion is valid. [9]

10. The matrix A is given by $A = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}$.

   (a) Find the matrix B such that $B = A^2 - 6A + 11I$, where I is the 3×3 identity matrix. [3]

   (b) Find AB and deduce A^{-1}. [3]

   (c) Hence, solve the following simultaneous equations,

   $x - z = 2$, $x + 2y + z = 5$ and $2x + 2y + 3z = 1$. [5]

11. (a) The roots of the quadratic equation $z^2 + z + 1 = 0$ are denoted by z_1 and z_2.

   (i) Find z_1 and z_2 in the form $a + bi$ where $a, b \in \mathcal{R}$. [3]

   (ii) Verify that $(z_2)^2 = z_1$ and $(z_1)^2 = z_2$. [2]

   (b) In the Argand diagram, the points P_1 and P_2 represent the complex numbers z and z^2 respectively, where $z = \sqrt{2} + \sqrt{2}\,i$.

   (i) Find the modulus and argument of z. [2]

   (ii) Find the area of the triangle OP_1P_2, where O is the origin. [3]

12. Given the curve $y = \frac{(2x+1)^2}{4x(1-x)}$.

   (a) State equations of the asymptotes.

   (b) Find the coordinates of the turning points.

   Sketch the curve showing the asymptotes and turning points. [8]

   The line $y = x$ and the curve $y = \frac{(2x+1)^2}{4x(1-x)}$ intersect at point A, whose x coordinate is α.

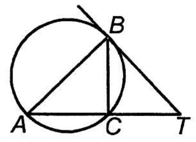
   Show that α is a root of the equation $4x^3 + 4x + 1 = 0$, and α lies in the interval $-0.25 < \alpha < 0$.

   By taking -0.25 as the first approximation to α, use the Newton–Raphson method to find the root α, giving your answer correct to 3 decimal places. [7]

NEGERI SEMBILAN (2006)
MATHEMATICS T (PAPER 2)

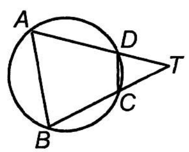
1. If $x - y = z$, express $\frac{dy}{dx}$ in terms of $\frac{dz}{dx}$.
Using the substitution $x - y = z$, solve the differential equation
$\frac{dy}{dx} = x - y$, given that $y = 0$ when $x = 0$. [6]

2. In the figure, BT is a tangent to the circle ABC, and ACT is a straight line.
If $\angle ABC = \angle BTA$, prove that AB is diameter. [6]



3. A raindrop falls with the rate of change of velocity $(9.8 - \frac{v}{3})$ ms^{-2}, where v is its velocity.
Show that the raindrop's velocity approaches a limiting value of 29.4 ms^{-1}. [5]

4. In the figure, two chords of the circle when produced, meet at T.
Given that AD = 6 cm, DT = 4 cm and TB = 9 cm.



(a) Prove that triangles TCD and TAB are similar.
(b) Find the length of TC.
(c) Find the ratio of the area of ΔTDC : area of ΔTAB. [10]

5. (a) Find the values of x, in the interval $0^\circ \le x \le 360^\circ$, that satisfy the equation
$\cos(x - 30^\circ) = 2\sin x$. [4]
(b) Express $12\cos x - 5\sin x$ in the form of $r\cos(x + \alpha)$, where r is positive and α is an acute angle. Hence, determine the value of x for $0^\circ \le x \le 360^\circ$ for which $12\cos x - 5\sin x$ is maximum. [7]

6. A car, A is travelling with velocity $(50\underline{i} + 40\underline{j})$ km h^{-1}, while another car B, is travelling with velocity $(20\underline{i} + 30\underline{j})$ km h^{-1}, where $\underline{i}$ and $\underline{j}$ are unit vectors eastwards and northwards respectively.
(a) Calculate the magnitude of the velocity of A relative to B.
(b) At the instant, when A is exactly east of B, A changes its direction but maintains its speed in order to intercept B. Find the direction A must be driven. [12]

7. The continuous random variable X has a probability distribution density function $f(x)$
given by $f(x) = \begin{cases} k\left(\frac{2}{x^2} - \frac{1}{2}\right) & , 1 \le x \le 2 \\ 0 & ,\text{otherwise} \end{cases}$, where k is a constant.
(a) Show that $k = 2$.
(b) Find the mean of X. [5]

8. The continuous random variable X is the distance measured in hundreds of kilometers, that a particular car will travel on a full tank of petrol. It is given that
$$P(X \le x) = \begin{cases} 0 & , x < 3 \\ ax^2 - 8ax + b & , 3 \le x \le 4 \\ 1 & , 4 < x \end{cases}$$
, where a and b are constants.
(a) Show that $a = -1$.
(b) Find the values of b.
(c) Verify that $P(X \le 3.5) = \frac{3}{4}$. [5]

9. Two events A and B are such that $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{2}$.
Calculate $P(A' \cap B)$ in each of the cases when
(a) $P(A \cap B) = \frac{1}{8}$, [2]
(b) A and B are mutually exclusive, [2]
(c) A is a subset of B. [2]

10. The probability that Hamid will pass his driving test is $\frac{7}{10}$ and the probability that Wan will pass her driving text is $\frac{1}{2}$. The probability that Hamid will pass the test, given that Wan had passed her test, is $\frac{14}{15}$. Find the probability that
(a) both of them pass the driving test, [3]
(b) only one of them pass the test. [3]

11. 50 mice were tested on their capability in finishing an obstacle course.
The time taken, in seconds are as follows :

15	57	25	24	48	25	29	37	26	54
21	37	51	52	29	33	35	26	18	39
36	30	67	19	36	41	24	45	42	27
42	56	23	34	35	63	16	57	37	22
31	43	32	13	53	28	31	35	46	47

(a) Construct a frequency table with the class size of 10 seconds, the first class being 10 – 19. [2]
(b) Estimate the mean and variance. [5]
(c) Draw a histogram based on the frequency table.
Hence, estimate the mode and median. [6]

12. In a large city, 1 person in 5 wears spectacles.
(a) Find the probability that in a random sample of 10 people
(i) exactly 4 people wear spectacles,
(ii) less than half of them wear spectacles.
(b) Find the mean and the standard deviation of the number of the people who wear spectacles in a random sample of 40 people.
(c) Determine the minimum size of a random sample so that the probability that it contains at least one person who wear spectacles is greater than 0.90. [15]

1. $x\log_8 32 - y\log_4 2 = \frac{3}{2}$

$\therefore x\left[\frac{5\lg 2}{3\lg 2}\right] - y\left[\frac{\lg 2}{2\lg 2}\right] = \frac{3}{2} = \frac{5x}{3} - \frac{y}{2}$

$\therefore 9 = 10x - 3y$ — ①

$\log_2 x + \frac{1}{2}\log_2 y^2 = 2\log_4 3$

$\therefore \log_2 x + \log_2 y = 2\left[\frac{\log_2 3}{2\log_2 2}\right]$

$\log_2(xy) = \log_2 3 \Rightarrow xy = 3$ — ②

②→①: $9 = 10x - 3\left(\frac{3}{x}\right) \Rightarrow 10x^2 - 9x - 9 = 0$

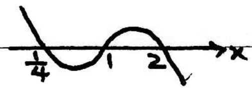
$(5x+3)(2x-3) = 0 \Rightarrow x = \frac{3}{2}$ [∵ $x>0$]

$\therefore x = \frac{3}{2}, y = 2$ ※

2. $\frac{3}{1-x} \le 5-4x \Rightarrow 3(1-x) \le (5-4x)(1-x)^2$

$\therefore (1-x)[(5-4x)(1-x)-3] \ge 0$ [$x \ne 1$]

$\therefore (1-x)(4x-1)(x-2) \ge 0$



$\therefore \{x : x \le \frac{1}{4}$ or $1 < x \le 2\}$ ※

3. $f(x) = \frac{x^2-3x}{x^2-2x+1} = 1 - \frac{x+1}{(x-1)^2}$

$= 1 - \left[\frac{(x-1)+2}{(x-1)^2}\right] = 1 - \frac{1}{x-1} - \frac{2}{(x-1)^2}$ ※

4.(a) $S_3 = 6$: $3p + 9q = 6 \Rightarrow p + 3q = 2$ — ①

$S_5 = 11$: $5p + 25q = 11 \Rightarrow 5(2-3q) + 25q = 11$

$\therefore q = 0.1$ $\therefore p = 1.7$ ※

(b) $T_n = [1.7n + 0.1n^2] - [1.7(n-1) + 0.1(n-1)^2]$

$= 0.2n + 1.6$ ※

$d = T_{n+1} - T_n = [0.2(n+1) + 1.6] - [0.2n + 1.6]$

$\therefore d = 0.2$ ※

5.(a) $\int \frac{x}{(x+1)^2}\,dx = \int \left[\frac{x+1-1}{(x+1)^2}\right] dx$

$= \int \left[\frac{1}{x+1} - \frac{1}{(x+1)^2}\right] dx = \ln(x+1) + \frac{1}{x+1} + c$ ※

(b) $\int_0^4 \frac{1}{1+\sqrt{x}}\,dx \approx \frac{1}{2}(1)\left[1 + 2\left(\frac{1}{2} + \frac{1}{1+\sqrt{2}} + \frac{1}{1+\sqrt{3}}\right) + \frac{1}{3}\right]$

$= 1.95$ (3s.f) ※

6.(a) $x^2 + y^2 - 18x - 6y + 45 = 0$

$\therefore (x-9)^2 - 81 + (y-3)^2 - 9 + 45 = 0$

$\therefore (x-9)^2 + (y-3)^2 = 45$

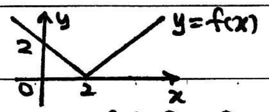
$\therefore C(9,3)$, $r = \sqrt{45} = 3\sqrt{5}$ ※

(b) $d_c = \frac{|9-3|}{\sqrt{1+1}} = 3\sqrt{2}$ ※ $(= CN)$

(c) $ON^2 = OC^2 - NC^2 = (9^2+3^2) - 18 = 72$

$\therefore$ Area $= \frac{1}{2}(3\sqrt{2})(6\sqrt{2}) = 18$ ※

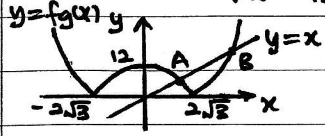
7.(a)



f^{-1} does not exist because f is not one to one. ※

(b) $fg(x) = f(x^2-10) = |x^2-10-2|$

$= |x^2-12|, x \in \mathbb{R}$. ※



For A: $-(x^2-12) = x$

$x^2 + x - 12 = 0$

$\therefore (x-3)(x+4) = 0$

[∵ $x_A > 0$]: $x_A = 3$

For B: $x^2 - 12 = x$

$x^2 - x - 12 = 0$

$(x+3)(x-4) = 0$

[∵ $x_B > 0$]: $x_B = 4$

$\therefore fg(x) = x$

$\Rightarrow x = 3$ or 4 ※

8. $f(2) = 8 - 38 + 30 = 0$

$\therefore (x-2)$ is a factor of $f(x)$ ※

$\therefore f(x) = x^3 - 19x + 30 = (x-2)(x^2+2x-15)$

$= (x-2)(x-3)(x+5)$ ※

$f(x) = 2(x-2)$: $(x-2)[x^2+2x-15-2] = 0$

$x = 2$ or $x = \frac{-2 \pm \sqrt{4+4(17)}}{2}$

$\therefore x = 2$ or $-1 \pm 3\sqrt{2}$ ※

9. $\frac{1}{(a+bx)^2} = \frac{1}{a^2}\left(1 + \frac{bx}{a}\right)^{-2}$

$= \frac{1}{a^2}\left[1 + (-2)\left(\frac{bx}{a}\right) + \frac{(-2)(-3)}{2!}\left(\frac{bx}{a}\right)^2 + \dots\right]$

$= \frac{1}{a^2} - \frac{2b}{a^3}x + \frac{3b^2}{a^4}x^2 + \dots$ ※

coef of x = coef of x^2: $-\frac{2b}{a^3} = \frac{3b^2}{a^4}$

$\frac{b}{a^4}[3b + 2a] = 0$

[∵ $a \ne 0, b \ne 0$]: $3b + 2a = 0$ ※, $\left(\frac{b}{a} = -\frac{2}{3}\right)$

$\frac{1}{a^2}\left[1 + \frac{3b^2}{a^2}\right] = 84 \Rightarrow 1 + 3\left(\frac{4}{9}\right) = 84a^2$

$\therefore a^2 = \frac{1}{36} \Rightarrow a = \pm\frac{1}{6}$

$a = \frac{1}{6}: b = -\frac{1}{9}$; $a = -\frac{1}{6}: b = \frac{1}{9}$ ※

For Valid Exp: $\left|\frac{b}{a}x\right| < 1 \Rightarrow |x| < \frac{3}{2}$

$\{x : -\frac{3}{2} < x < \frac{3}{2}\}$ ※

10. $A^2 = \begin{pmatrix} 1+0-2 & 0+0-2 & -1+0-3 \\ 1+2+2 & 0+4+2 & -1+2+3 \\ 2+2+6 & 0+4+6 & -2+2+9 \end{pmatrix} = \begin{pmatrix} -1 & -2 & -4 \\ 5 & 6 & 4 \\ 10 & 10 & 9 \end{pmatrix}$

(a) $B = A^2 - 6A + 11I = \begin{pmatrix} 4 & -2 & 2 \\ -1 & 5 & -2 \\ -2 & -2 & 2 \end{pmatrix}$ ※

(b) $AB = \begin{pmatrix} 4+0+2 & -2+0+2 & 2+0-2 \\ 4-2-2 & -2+10-2 & 2-4+2 \\ 8-2-6 & -4+10-6 & 4-4+6 \end{pmatrix} = \begin{pmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{pmatrix}$ ※

$\therefore AB = 6I \Rightarrow A^{-1} = \frac{1}{6}B = \frac{1}{6}\begin{pmatrix} 4 & -2 & 2 \\ -1 & 5 & -2 \\ -2 & -2 & 2 \end{pmatrix}$

(c) $\left.\begin{array}{l} x - z = 2 \\ x + 2y + z = 5 \\ 2x + 2y + 3z = 1 \end{array}\right\}$ $A\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}\begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix}$

$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{6}\begin{pmatrix} 4 & -2 & 2 \\ -1 & 5 & -2 \\ -2 & -2 & 2 \end{pmatrix}\begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} = \frac{1}{6}\begin{pmatrix} 0 \\ 21 \\ -12 \end{pmatrix}$

$\therefore x = 0, y = \frac{7}{2}, z = -2$ ※

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11.(a) $z^2+z+1=0 \Rightarrow z=\frac{1}{2}[-1\pm\sqrt{1-4}]$ *

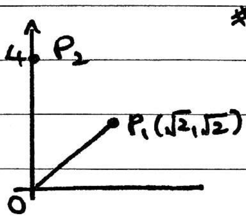
(i) $z_1=-\frac{1}{2}+\frac{\sqrt{3}}{2}i$, $z_2=-\frac{1}{2}-\frac{\sqrt{3}}{2}i$

(ii) $z_1^{\,2}=-1-z_1=-1-(-\frac{1}{2}+\frac{\sqrt{3}}{2}i)=-\frac{1}{2}-\frac{\sqrt{3}}{2}i=z_2$ *

$z_2^{\,2}=-1-z_2=-1-[-\frac{1}{2}-\frac{\sqrt{3}}{2}i]=-\frac{1}{2}+\frac{\sqrt{3}}{2}i=z_1$ *

(b) $z=\sqrt{2}(1+i)$:

$z^2=2(1-1+2i)=4i$



(i) $|z|=\sqrt{2}(1+1)^{1/2}=2$ *

$\arg(z)=\tan^{-1}(\frac{1}{1})=\frac{\pi}{4}$ *

(ii) Area $=4(\sqrt{2})(\frac{1}{2})=2\sqrt{2}$ *

12.(a) $y=\frac{(2x+1)^2}{4x(1-x)}=\frac{4x^2+4x+1}{4x-4x^2}=\frac{8x+1}{4x(1-x)}-1$

(i) asymptotes: $y=-1$, $x=0$, $x=1$ *

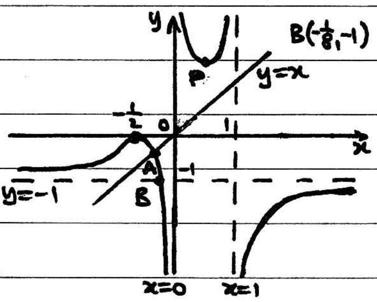
$\frac{dy}{dx}=\frac{4x(1-x)(8)-(8x+1)(4-8x)}{(4x-4x^2)^2}$

For $\frac{dy}{dx}=0$: $8x(1-x)-(8x+1)(1-2x)=0$

$\therefore 8x^2+2x-1=0$

$(4x-1)(2x+1)=0$

$\therefore x=\frac{1}{4}$ or $-\frac{1}{2}$



$\therefore$ turning points $=P(\frac{1}{4},3)$, $(-\frac{1}{2},0)$ *

(b)

(i) For A: $\frac{(2x+1)^2}{4x(1-x)}=x$

$4x^2+4x+1=4x^2-4x^3 \Rightarrow 4x^3+4x+1=0$

$\therefore \alpha$ is a root of $4x^3+4x+1=0$ *

(ii) Let $f(x)=4x^3+4x+1$: $f'(x)=12x^2+4$

$f(-\frac{1}{4}).f(0)=(-\frac{1}{16})(1)=-\frac{1}{16}$ (<0)

$\therefore -\frac{1}{4}<\alpha<0$ *

(iii) By NRM: $x_1=x-\frac{4x^3+4x+1}{12x^2+4}$

$x_0=-\frac{1}{4}$: $x_1=-0.23684 \approx -0.237$

$x_2=-0.23673 \approx -0.237$

$\therefore \alpha=-0.237$ (3dp) *

1. $x-y=z$: $1-\frac{dy}{dx}=\frac{dz}{dx} \Rightarrow \frac{dy}{dx}=1-\frac{dz}{dx}$ *

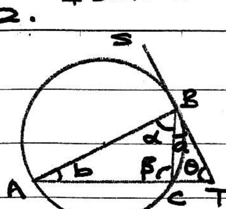
$\frac{dy}{dx}=x-y$: $1-\frac{dz}{dx}=z \Rightarrow \int dx=\int\frac{dz}{1-z}$

$x=-\ln(1-z)+c=-\ln(1-x+y)+c$

$x=0, y=0$: $0=c$

$\therefore e^{-x}=1-x+y \Rightarrow y=x-1+e^{-x}$ *

2. $\angle BCT=z$



$a=b$ [∠ of alternate segment]

$\alpha=\theta$ [given]

$\therefore z=b+\alpha$ [Ext. ∠ of Δ]

$=a+\theta$

$=\beta$ [Ext. ∠ of Δ]

$\beta+z=180°$ [supplementary ∠]

$\therefore \beta=90°$

$\therefore \beta$ is an angle (90°) in semicircle

$\therefore$ AB is a diameter. *

3. $\frac{dv}{dt}=9.8-\frac{v}{3} \Rightarrow \int\frac{dv}{29.4-v}=\int\frac{1}{3}dt$

$\therefore -\ln(29.4-v)=\frac{1}{3}t+c$ — ①

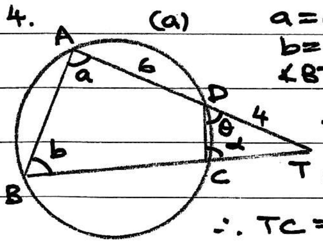
$t=0, v=0$: $-\ln 29.4=c$ — ②

$\therefore$ ② − ①: $\ln\left(\frac{29.4-v}{29.4}\right)=-\frac{1}{3}t$

$\therefore v=29.4[1-e^{-\frac{1}{3}t}]$

$\lim_{t\to\infty} v=29.4[1-0]=29.4$ *

4.



(a) $a=\alpha$ [Ext. ∠ of cyclic ABCD]

$b=\theta$ [Ext ∠ of cyclic ABCD]

$\angle BTA=\angle DTC$ [common ∠]

$\therefore \Delta TCD \sim \Delta TAB$ *

(b) $\frac{TC}{TA}=\frac{TD}{TB}=\frac{4}{9}$

$\therefore TC=10(\frac{4}{9})=\frac{40}{9}$ cm *

(c) $\frac{\text{area of } \Delta TDC}{\text{area of } \Delta TAB}=\left(\frac{TD}{TB}\right)^2=\frac{16}{81}$ *

5.(a) $\cos(x-30°)=2\sin x=\frac{\sqrt{3}}{2}\cos x+\frac{1}{2}\sin x$

$\therefore \tan x=\frac{\sqrt{3}}{3} \Rightarrow x=30°, 210°$ *

(b) $f(x)=12\cos x-5\sin x=r\cos(x+\alpha)$

$=r\cos x\cos\alpha-r\sin x\sin\alpha$

$\therefore r\cos\alpha=12$, $r\sin\alpha=5$ | $r(\frac{5}{13})=5$

$\therefore \tan\alpha=\frac{5}{12} \Rightarrow \alpha=22.6°$ | $\therefore r=13$

$\therefore f(x)=13\cos(x+22.6°)$ *

$f(x)$ max when $\cos(x+22.6°)=1$

$\therefore x+22.6°=360° \Rightarrow x=337.4°$ *

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No.: .... Date: ....

6. (a) $\underline{V}_{AB} = \begin{pmatrix}50\\40\end{pmatrix} - \begin{pmatrix}20\\30\end{pmatrix} = \begin{pmatrix}30\\10\end{pmatrix} = 10\begin{pmatrix}3\\1\end{pmatrix}$

$= 10\sqrt{9+1}\ kmh^{-1}$, to $N \tan^{-1}(\frac{3}{1})E$

$= 10\sqrt{10}\ kmh^{-1}$, to $N\ 71.57°E$ *

(b) To intercept: $\underline{V}_{AB}$ (to west) $= k\begin{pmatrix}-1\\0\end{pmatrix}$

Let $\underline{V}_A = \begin{pmatrix}a\\b\end{pmatrix}$: $\begin{pmatrix}a\\b\end{pmatrix} - \begin{pmatrix}20\\30\end{pmatrix} = \begin{pmatrix}-k\\0\end{pmatrix}$ $(k>0)$

$\therefore b = 30$ and $k = 20 - a$ $(a < 20)$

$|\underline{V}_A| = 10\sqrt{41}$: $a^2 + 30^2 = 4100 \Rightarrow a = \pm 40\sqrt{2}$

$(\because a < 20)$: $a = -40\sqrt{2}$

$\therefore$ Direction of $\underline{V}_A = N \tan^{-1}(\frac{4\sqrt{2}}{3})W$

$= N\ 62.06°W$ *

7. (a) $\int_1^2 k[\frac{2}{x^2} - \frac{1}{2}]dx = 1 = k[-\frac{2}{x} - \frac{x}{2}]_1^2$

$\therefore k[(-1-1)-(-2-\frac{1}{2})] = 1 \Rightarrow k = 2$ *

(b) $E(X) = \int_1^2 x \cdot 2(\frac{2}{x^2} - \frac{1}{2})dx$

$= \int_1^2 [\frac{4}{x} - x]dx = [4\ln x - \frac{x^2}{2}]_1^2$

$= [4\ln 2 - 2] - [0 - \frac{1}{2}] = 4\ln 2 - \frac{3}{2}$ *

8. (a) $x=3$: $0 = 9a - 24a + b \Rightarrow b = 15a$

$x=4$: $16a - 32a + b = 1 \Rightarrow -16a + 15a = 1$

$\therefore a = -1$ *

(b) $\therefore b = -15$ *

(c) $P(X \leq 3.5) = -(3.5)^2 + 8(3.5) - 15 = \frac{3}{4}$ *

9. (a) $P(A' \cap B) = P(B) - P(A \cap B)$

$= \frac{1}{2} - \frac{1}{8} = \frac{3}{8}$ *

(b) $P(A' \cap B) = \frac{1}{2} - 0 = \frac{1}{2}$ *

(c) $P(A' \cap B) = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$ *

10. $P(H) = \frac{7}{10}$, $P(W) = \frac{1}{2}$

(a) $P(H|W) = \frac{14}{15} = \frac{P(H \cap W)}{1/2}$

$\therefore P(H \cap W) = \frac{7}{15}$ *

(b) $P(\text{only one pass}) = \frac{7}{10} + \frac{1}{2} - 2(\frac{7}{15})$

$= \frac{4}{15}$ *

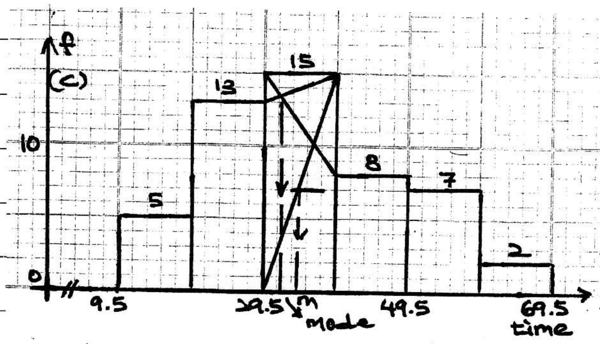
11. (a) $n = 50$, $\Sigma ft = 1775$, $\Sigma ft^2 = 71662.5$

time	f	Midpoint (t)
$9.5 \leq t < 19.5$	5	14.5
$19.5 \leq t < 29.5$	13	24.5
$29.5 \leq t < 39.5$	15	34.5
$39.5 \leq t < 49.5$	8	44.5
$49.5 \leq t < 59.5$	7	54.5
$59.5 \leq t < 69.5$	2	64.5

$\bar{t} = \frac{1775}{50} = 35.5$ *

$s^2 = \frac{\Sigma ft^2}{n} - (\bar{t})^2$

$= 173$ *



Mode $= 29.5 + 1.2(2) = 31.9$ *

$m = 29.5 + 2.3(2) = 34.1$ *

12. Let X = no. of person wear specs.

(a) $X \sim B(10, 0.2)$:

(i) $P(X=4) = {}^{10}C_4(0.2)^4(0.8)^6 = 0.0881$ *

(ii) $P(X<5) = 0.8^{10} + 10(0.2)0.8)^9$

$+ {}^{10}C_2(0.2)^2(0.8)^8 + {}^{10}C_3(0.2)^3(0.8)^7$

$+ {}^{10}C_4(0.2)^4(0.8)^6 = 0.9672$ *

(b) $X \sim B(40, 0.2)$: $E(X) = 40(0.2) = 8$ *

$s_x = \sqrt{40(0.2)(0.8)} = 2.53$ *

(c) $X \sim B(n, 0.2)$: $P(X \geq 1) > 0.9$

$\therefore P(X=0) < 0.1 \Rightarrow 0.8^n < 0.1$

$\therefore n > \frac{\lg 0.1}{\lg 0.8}$, $[\because \lg 0.8 < 0]$

$\therefore n > 10.32$

$\therefore n_{min} = 11$ *

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