## STPM 2006
## MATHEMATICS T (PAPER 1)

1. If $A$, $B$ and $C$ are arbitrary sets, show that $[(A \cup B) - (B \cup C)] \cap (A \cup C)' = \phi$. [4]

2. If $x$ is so small that $x^2$ and higher powers of $x$ may be neglected, show that
$$(1-x)^6\left(2+\frac{x}{2}\right)^{10} \approx 2^9(2-7x).$$ [4]

3. Determine the values of $k$ such that the determinant of the matrix
$$\begin{pmatrix} k & 1 & 3 \\ 2k+1 & -3 & 2 \\ 0 & k & k \end{pmatrix}$$ is 0. [4]

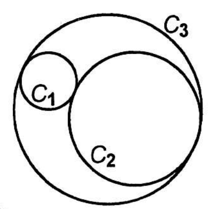
4. Using the trapezium rule, with five ordinates, evaluate $\int_0^1 \sqrt{4-x^2}\,dx$. [4]

5. If $y = x\ln(x+1)$, find an approximation for the increase in $y$ when $x$ increases by $\delta x$.
Hence, estimate the value of ln 2.01 given that ln 2 = 0.6931. [6]

6. Express $\dfrac{2x+1}{(x^2+1)(2-x)}$ in the form $\dfrac{Ax+B}{x^2+1} + \dfrac{C}{2-x}$, where $A$, $B$ and $C$ are constants. [3]
Hence, evaluate $\displaystyle\int_0^1 \frac{2x+1}{(x^2+1)(2-x)}\,dx$. [4]

7. The $n^{th}$ term of an arithmetic progression is $T_n$.
Show that $U_n = \frac{2}{5}(-2)^{2\left(\frac{10-T_n}{17}\right)}$ is the $n^{th}$ term of a geometric progression. [4]
If $T_n = \frac{1}{2}(17n - 14)$, evaluate $\displaystyle\sum_{n=1}^{\infty} U_n$. [4]

8. Show that $x^2 + y^2 - 2ax - 2by + c = 0$ is the equation of the circle with centre $(a, b)$ and radius $\sqrt{a^2+b^2-c}$. [3]




The figure shows three circles $C_1$, $C_2$ and $C_3$ touching one another, where their centres lie on a straight line.
If $C_1$ and $C_2$ have equations $x^2 + y^2 - 10x - 4y + 28 = 0$ and $x^2 + y^2 - 16x + 4y + 52 = 0$ respectively, find the equation of $C_3$. [7]

9. Functions $f$, $g$ and $h$ are defined by
$$f : x \to \frac{x}{x+1},\quad g : x \to \frac{x+2}{x},\quad h : x \to 3 + \frac{2}{x}.$$
(a) State the domains of $f$ and $g$. [2]
(b) Find the composite function $g \circ f$ and state its domain and range. [5]
(c) State the domain and range of $h$. [2]
(d) State whether $h = g \circ f$. Give reason for your answer. [2]

10 The polynomial $p(x) = x^4 + ax^3 - 7x^2 - 4ax + b$ has a factor $x + 3$ and, when divided by $x - 3$, has remainder 60.
Find the values of $a$ and $b$, and factorise $p(x)$ completely. [9]
Using the substitution $y = \frac{1}{x}$, solve the equation $12y^4 - 8y^3 - 7y^2 + 2y + 1 = 0$. [3]

11. If $\mathbf{P} = \begin{pmatrix} 5 & 2 & 3 \\ 1 & -4 & 3 \\ 3 & 1 & 2 \end{pmatrix}$, $\mathbf{Q} = \begin{pmatrix} a & 1 & -18 \\ b & -1 & 12 \\ -13 & -1 & c \end{pmatrix}$ and $\mathbf{PQ} = 2\mathbf{I}$, where $\mathbf{I}$ is the 3 x 3 identity matrix, determine the values of $a$, $b$ and $c$. Hence, find $\mathbf{P}^{-1}$. [8]
Two groups of workers have their drinks at a stall. The first group comprising ten workers have five cups of tea, two cups of coffee and three glasses of fruit juice at a total cost of RM 11.80. The second group of six workers have three cups of tea, a cup of coffee and two glasses of fruit juice at a total cost of RM 7.10. The cost of a cup of tea and three glasses of fruit juice is the same as the cost of four cups of coffee. If the cost of a cup of tea, a cup of coffee and a glass of fruit juice are RM $x$, RM $y$ and RM $z$ respectively, obtain a matrix equation to represent the above information. Hence, determine the cost of each drink. [6]

12. The function $f$ is defined by $f(t) = \dfrac{4e^{kt}-1}{4e^{kt}+1}$, where $k$ is a positive constant.
(a) Find the value of $f(0)$. [1]
(b) Show that $f'(t) > 0$. [5]
(c) Show that $k\{1 - [f(t)]^2\} = 2f'(t)$ and, hence, show that $f''(t) < 0$, for $t > 0$. [6]
(d) Find $\lim_{t\to\infty} f(t)$. [2]
(e) Sketch the graph of $f$. [2]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**MATHEMATICS T (PAPER 2)**

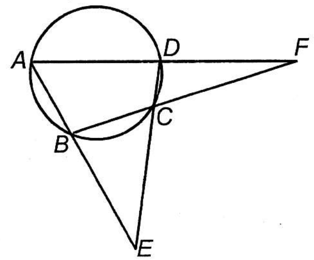
1. Express $4 \sin \theta - 3 \cos \theta$ in the form $r \sin(\theta - \alpha)$, where $r > 0$ and $0^\circ < \alpha < 90^\circ$.
Hence, solve the equation $4 \sin \theta - 3 \cos \theta = 3$ for $0^\circ < \theta < 360^\circ$. [6]

2. If the angle between the vectors $\underline{a} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$ and $\underline{b} = \begin{pmatrix} 1 \\ p \end{pmatrix}$ is $135^\circ$, find the value of $p$. [6]

3. Find the general solution of the differential equation $x\frac{dy}{dx} = y^2 - y - 2$. [6]

4. The points $P$, $Q$ and $R$ are the midpoints of the sides $BC$, $CA$ and $AB$ respectively of the triangle $ABC$. The lines $AP$ and $BQ$ meet at the point $G$, where $AG = m\,AP$ and $BG = n\,BQ$.
(a) Show that $\overrightarrow{AG} = \frac{1}{2} m \overrightarrow{AB} + \frac{1}{2} m \overrightarrow{AC}$ and $\overrightarrow{AG} = (1 - n)\overrightarrow{AB} + \frac{1}{2} n \overrightarrow{AC}$.
Deduce that $AG = \frac{2}{3} AP$ and $BG = \frac{2}{3} BQ$. [6]
(b) Show that $CR$ meets $AP$ and $BQ$ at $G$, where $CG = \frac{2}{3} CR$. [3]

5. Prove that an exterior angle of a cyclic quadrilateral is equal to the opposite interior angle. [3]
In the diagram, $ABCD$ is a cyclic quadrilateral. The lines $AB$ and $DC$ extended meet at the point $E$ and the lines $AD$ and $BC$ extended meet at the point $F$. Show that the triangles $ADE$ and $CBE$ are similar. [3]




If $DA = DE$, $\angle CFD = \alpha$ and $\angle BEC = 3\alpha$, determine the value of $\alpha$. [4]

6. A particle moves from rest along a horizontal straight line. At time $t$ s, the displacement and velocity of the particle are $x$ m and $v$ ms$^{-1}$ respectively and its acceleration, in ms$^{-2}$, is given by $\frac{dv}{dt} = \sin \pi t - \sqrt{3} \cos \pi t$.
Express $v$ and $x$ in terms of $t$. [7]
Find the velocities of the particle when its acceleration is zero for the first and second times.
Find also the distance travelled by the particle between the first and second times its acceleration is zero. [7]

2

7. Two archers $A$ and $B$ take turns to shoot, with archer $A$ taking the first shot. The probabilities of archers $A$ and $B$ hitting the bull's-eye in each shot are $\frac{1}{6}$ and $\frac{1}{5}$ respectively. Show that the probability of archer $A$ hitting the bull's-eye first is $\frac{1}{2}$. [4]

8. The probability that it rains in a certain area is $\frac{1}{5}$. The probability that accident occurs at a particular corner of a road in that area is $\frac{1}{20}$ if it rains and $\frac{1}{50}$ if it does not rain. Find the probability that it rains if an accident occurs at the corner. [5]

9. The independent Poisson random variables $X$ and $Y$ have parameters 0.5 and 3.5 respectively. The random variable $W$ is defined by $W = X - Y$.
(a) Find E($W$) and Var($W$). [4]
(b) Give one reason why $W$ is not a Poisson random variable. [1]

10. The probability that a heart patient survives after surgery in a country is 0.85.
(a) Find the probability that, out of five randomly chosen heart patients undergoing surgery, four survive. [3]
(b) Using a suitable approximate distribution, find the probability that more than 160 survive after surgery in a random sample of 200 heart patients. [6]

11. The times taken by 22 students to breakfast are shown in the table.

| Time ($x$ minutes) | No. of students |
|---|---|
| $2 \le x < 5$ | 1 |
| $5 \le x < 8$ | 2 |
| $8 \le x < 11$ | 4 |
| $11 \le x < 14$ | 8 |
| $14 \le x < 17$ | 5 |
| $17 \le x < 20$ | 2 |

(a) Draw a histogram of the grouped data.
Comment on the shape of the frequency distribution. [4]
(b) Calculate estimates of the mean, median and mode of the breakfast times.
Use your calculations to justify your statement about the shape of the frequency distribution. [7]

12. The continuous random variable $X$ has probability density function

$$f(x) = \begin{cases} \sqrt{\dfrac{x-1}{12}} & ,1 \le x \le b \\ 0 & ,\text{otherwise} \end{cases}, \text{ where } b \text{ is a constant.}$$

(a) Determine the value of $b$. [4]
(b) Find the cumulative distribution function of $X$ and sketch its graph. [5]
(c) Calculate E($X$). [6]

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1. $[(A\cup B)-(B\cup C)]\cap(A\cup C)'$

$=[(A\cup B)\cap(B\cup C)']\cap(A'\cap C')$

$=(A\cup B)\cap[(B'\cap C')\cap(A'\cap C')]$

$=(B\cup A)\cap[(B'\cap A')\cap(C'\cap C')]$

$=[(B\cup A)\cap(B\cup A)']\cap C'$

$=\emptyset\cap C' = \emptyset$ $\#$

2. $(1-x)^6(2+\frac{x}{2})^{10}$

$=[1+6(-x)+\dots][2^{10}+10(2^9)(\frac{x}{2})+\dots]$

$=2^{10}+[5(2^9)-6(2^{10})]x+\dots$

$\approx 2^9[2-7x]$ $\#$

3. det $=0$: $k(-6-2k)-(2k+1)(2-3k)=0$

$\therefore 4k^2-7k-2=0$

$\therefore (4k+1)(k-2)=0 \Rightarrow k=-\frac{1}{4}$ or $2$ $\#$

4. $\int_0^1\sqrt{4-x^2}\,dx \approx \frac{1}{2}(\frac{1}{4})[2+2(\frac{3}{4}\sqrt{7} + \frac{1}{2}\sqrt{15}+\frac{1}{4}\sqrt{55})+\sqrt{3}]=1.910$ (3dp) $\#$

5. $y=x\ln(x+1)\Rightarrow y'=\ln(x+1)+\frac{x}{x+1}$

$\delta y\approx\delta x(\frac{dy}{dx})=\delta x[\ln(x+1)+\frac{x}{x+1}]$ $\#$

$x=1$, $\delta x=0.01$:

$\delta y \approx 0.01[\ln 2+\frac{1}{2}]=0.011931$

$\therefore (1.01)\ln 2.01\approx 0.6931+0.011931$

$\therefore \ln 2.01\approx 0.6981$ $\#$

6. $f(x)=\frac{2x+1}{(x^2+1)(2-x)}\equiv\frac{Ax+B}{x^2+1}+\frac{C}{2-x}$

$\equiv\frac{(Ax+B)(2-x)+C(x^2+1)}{(x^2+1)(2-x)}$

$\therefore 2x+1\equiv(Ax+B)(2-x)+C(x^2+1)$

$x=2$: $5=C(5)\Rightarrow C=1$

$[x^2]$: $0=-A+C\Rightarrow A=1$

$[x^0]$: $1=2B+C\Rightarrow B=0$

$\therefore f(x)=\frac{x}{x^2+1}+\frac{1}{2-x}$ $\#$

$\int_0^1 f(x)\,dx=[\frac{1}{2}\ln(x^2+1)-\ln(2-x)]_0^1$

$=(\frac{1}{2}\ln 2-0)-(0-\ln 2)=\frac{3}{2}\ln 2$ $\#$

7. $\frac{U_{n+1}}{U_n}=\frac{\frac{5}{2}(-2)^{2(\frac{10-T_{n+1}}{17})}}{\frac{5}{2}(-2)^{2(\frac{10-T_n}{17})}}$

$=(-2)^{\frac{2}{17}(10-T_{n+1}-10+T_n)}=(-2)^{-\frac{2}{17}d}$

($\because T_n$ is an AP $\Rightarrow d$ is a constant)

$\therefore r=-2^{-\frac{2}{17}d}$ is also a constant

$\therefore U_n$ is the nth term of a GP $\#$

$T_n=\frac{1}{2}(17n-14)$: $d=T_{n+1}-T_n$

$\therefore d=\frac{1}{2}(17(n+1)-14-17n+14)=\frac{17}{2}$

$\therefore r=(-2)^{-1}=-\frac{1}{2}$, $T_1=\frac{3}{2}$

$\therefore U_1=\frac{5}{2}(-2)^{\frac{2}{17}(10-\frac{3}{2})}=-5$

$\therefore \sum_{n=1}^{\infty}U_n=\frac{U_1}{1-r}=\frac{-5}{1+\frac{1}{2}}=-\frac{10}{3}$ $\#$

8. Eq: $(x-a)^2+(y-b)^2=(\sqrt{a^2+b^2-c})^2$

$\therefore x^2+y^2-2ax-2by+c=0$ $\#$

$C_1$: $x^2+y^2-10x-4y+28=0$

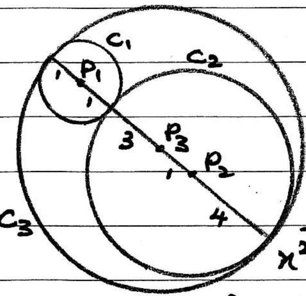
$\therefore$ Centre$(P_1)=(\frac{-10}{-2},\frac{-4}{-2})=(5,2)$

$\therefore r_1=\sqrt{5^2+2^2-28}=1$

$C_2$: $x^2+y^2-16x+4y+52=0$

$\therefore$ Centre$(P_2)=(\frac{-16}{-2},\frac{4}{-2})=(8,-2)$

$\therefore r_2=\sqrt{8^2+(-2)^2-52}=4$




$\therefore r_3=r_1+r_2=5$

$P_3=(\frac{1(5)+4(8)}{1+4},\frac{1(2)+4(-2)}{1+4})$

$=(\frac{37}{5},-\frac{6}{5})$

Eq: $(x-\frac{37}{5})^2+(y+\frac{6}{5})^2=5^2$

$x^2+y^2-\frac{74}{5}x+\frac{12}{5}y+\frac{156}{5}=0$

$\therefore 5x^2+5y^2-74x+12y+156=0$ $\#$

9. (a) $D_f$: $\{x\in\mathbb{R}: x\neq -1\}$; $D_g$: $\{x\in\mathbb{R}, x\neq 0\}$ $\#$

(b) $gf(x)=g[f(x)]=\frac{f(x)+2}{f(x)}$

$=\frac{x}{x+1}+2\Big/\frac{x}{x+1}=\frac{3x+2}{x}=3+\frac{2}{x}$

$\therefore gf: x\to 3+\frac{2}{x}$, $x\neq 0,-1$ $\#$

$D_{gf}$: $\{x\in\mathbb{R}: x\neq 0,-1\}$ $\#$, $R_{gf}$: $\{x\in\mathbb{R}: x\neq 1,3\}$ $\#$

(c) $D_h$: $\{x\in\mathbb{R}: x\neq 0\}$ $\#$

$R_h$: $\{x\in\mathbb{R}: x\neq 3\}$ $\#$

(d) $h\neq gf$ since $D_h\neq D_{gf}$ $\#$

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STPM 2006
MT1/MT2

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10. $P(-3)=0: 81-27a-63+12a+b=0$

$\therefore 18-15a+b=0$ —①

$P(3)=60: 81+27a-63-12a+b=60$

$\therefore 18+15a+b=60$ —②

②−①: $a=2$ $\therefore b=12$ ※

$\therefore P(x)=x^4+2x^3-7x^2-8x+12$

$P(1)=1+2-7-8+12=0$

$P(2)=16+16-28-16+12=0$ } $(x-1)$ and $(x-2)$ are factors of $P(x)$

$\therefore P(x)=(x-1)(x-2)(x+3)(x+2)$ ※

$y=\frac{1}{x}: 12y^4-8y^3-7y^2+2y+1=0$

$\therefore \frac{12}{x^4}-\frac{8}{x^3}-\frac{7}{x^2}+\frac{2}{x}+1=0$

$\therefore x^4+2x^3-7x^2-8x+12=0$

$\therefore x=1, 2, -3$ or $-2$ $=\frac{1}{y}$

$\therefore y=1, \frac{1}{2}, -\frac{1}{3}$ or $-\frac{1}{2}$ ※

11. $PQ=2I: (1,1): 5a+2b-39=2$

$\therefore 5a+2b=41$ —①

$(3,1): 3a+b-26=0$ —② } $a=11$, $b=-7$ ※

$(3,3): -54+12+2c=2 \Rightarrow c=22$ ※

$\therefore P^{-1}=\frac{1}{2}Q=\frac{1}{2}\begin{pmatrix}11 & 1 & -18\\ -7 & -1 & 12\\ -13 & -1 & 22\end{pmatrix}$ ※

$5x+2y+3z=11.80$
$3x+y+2z=7.10$
$x+3z=4y$
} $P\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}11.80\\0\\7.10\end{pmatrix}$

$\therefore \begin{pmatrix}x\\y\\z\end{pmatrix}=\frac{1}{2}\begin{pmatrix}11 & 1 & -18\\ -7 & -1 & 12\\ -13 & -1 & 22\end{pmatrix}\begin{pmatrix}11.80\\0\\7.10\end{pmatrix}=\begin{pmatrix}1\\1.3\\1.4\end{pmatrix}$

cost (tea) = RM1, cost (coffee) = RM1.30, cost (fruit juice) = RM1.40 ※

12. (a) $f(0)=\frac{4-1}{4+1}=\frac{3}{5}$ ※

(b) $f(t)=1-\frac{2}{4e^{kt}+1} \Rightarrow f'(t)=\frac{-2(-1)(4ke^{kt})}{(4e^{kt}+1)^2}$

$\therefore f'(t)=\frac{8ke^{kt}}{(4e^{kt}+1)^2}$

since $k>0, e^{kt}>0, 4e^{kt}+1>0$, $\forall t\in\mathbb{R}$

$\Rightarrow f'(t)>0$. ※

(c) $k(1-[f(t)]^2)=k[1-f(t)][1+f(t)]$

$=k\left[\frac{2}{4e^{kt}+1}\right]\left[2-\frac{2}{4e^{kt}+1}\right]$

$=k\left[\frac{2}{4e^{kt}+1}\right]\left[\frac{8e^{kt}}{4e^{kt}+1}\right]=2.f'(t)$

$\therefore 2.f''(t)=-k(2)[f(t)].f'(t)$

$\therefore f''(t)=-k.f(t).f'(t)$

For $t>0$: $[\because k>0, f'(t)>0]$

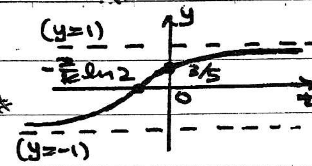
$\therefore e^{kt}>1 \Rightarrow 4e^{kt}-1>0,\ 4e^{kt}+1>0$

$\therefore f''(t)<0$, for $t>0$ ※

(d) $\lim_{t\to\infty} f(t)=1-\frac{2}{\infty}$

$=1-0=1$ ※

(e) $\lim_{t\to-\infty} f(t)=1-\frac{2}{0+1}=-1$




1. $f(\theta)=4\sin\theta-3\cos\theta=r\sin(\theta-\alpha)$

$=r\sin\theta\cos\alpha-r\cos\theta\sin\alpha$

$\therefore$ $r\cos\alpha=4$, $r\sin\alpha=3$ } $\tan\alpha=\frac{3}{4} \Rightarrow \alpha=36.87°$

$r(\frac{3}{5})=3 \Rightarrow r=5$

$\therefore f(\theta)=5\sin(\theta-36.87°)$ ※

$f(\theta)=3: \sin(\theta-36.87°)=\frac{3}{5}$

$\therefore \theta-36.87°=36.87°, 143.1°$

$\therefore \theta=73.7°$ or $180°$ ※

2. $\underline{a}\cdot\underline{b}=|\underline{a}||\underline{b}|\cos 135°$

$\therefore 4+8p=4\sqrt{5}.\sqrt{1+p^2}\left(-\frac{1}{\sqrt{2}}\right)$ — $(p<-\frac{1}{2})$

$\therefore 2(1+2p)^2=5(1+p^2)$

$\therefore 3p^2+8p-3=0 \Rightarrow (3p-1)(p+3)=0$

$[\because p<-\frac{1}{2}]$: $p=-3$ ※

3. $x\frac{dy}{dx}=y^2-y-2 \Rightarrow \int\frac{dy}{(y+1)(y-2)}=\int\frac{1}{x}dx$

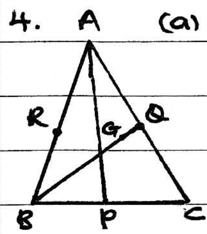
$\therefore \int\left[\frac{1}{y-2}-\frac{1}{y+1}\right]dy=\int\frac{3}{x}dx$

$\therefore \ln(y-2)-\ln(y+1)=3\ln x+c$

$\therefore \frac{y-2}{y+1}=Ax^3$ $[A=e^c]$

$\therefore 1-\frac{3}{y+1}=Ax^3 \Rightarrow y=\frac{3}{1-Ax^3}-1$ ※

4.




(a) $\vec{AG}=m\vec{AP}=m\left[\frac{1}{2}\vec{AB}+\frac{1}{2}\vec{AC}\right]$

$=\frac{m}{2}\vec{AB}+\frac{m}{2}\vec{AC}$ ※ —①

$\vec{AG}=n\vec{AQ}+(1-n)\vec{AB}$

$=n(\frac{1}{2}\vec{AC})+(1-n)\vec{AB}$

$=(1-n)\vec{AB}+\frac{n}{2}\vec{AC}$ ※ —②

①=②: $m=n$ and $\frac{m}{2}=1-n$

$\therefore \frac{n}{2}=1-n \Rightarrow n=\frac{2}{3}$ $\therefore m=\frac{2}{3}$

$\therefore AG=\frac{2}{3}AP$ and $BG=\frac{2}{3}BQ$ ※

(b) $\vec{CR}=\frac{1}{2}\vec{CB}+\frac{1}{2}\vec{CA} \Rightarrow 2\vec{CR}=\vec{CB}+\vec{CA}$

$\vec{CG}=\frac{1}{3}\vec{CB}+\frac{2}{3}\vec{CQ}=\frac{1}{3}\vec{CB}+\frac{2}{3}(\frac{1}{2}\vec{CA})$

$=\frac{1}{3}[\vec{CB}+\vec{CA}]=\frac{2}{3}\vec{CR}$

$\therefore CG // CR \Rightarrow C, G, R$ are collinear

$\Rightarrow$ CR meets AP, BQ at G, $CG=\frac{2}{3}CR$ ※

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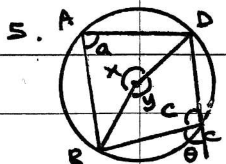
STPM 2006

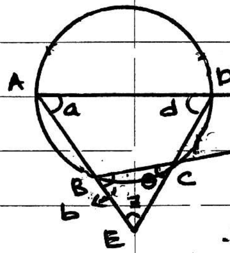
MT2

No.: ........................ Date: ........................

5. 


$y = 2a$ [Angle at centre]
$x = 2c$ [ " ]
$x + y = 360°$ ($\Sigma$ of $\angle$ about point)
$\therefore a + c = 180°$
$c + \theta = 180°$ [$\Sigma$ of $\angle$ on a line]
$\therefore a = \theta$

$\therefore$ Ext. angle of a cyclic quadrilateral is equal to the opposite interior angle *




$\left.\begin{array}{l} d = b \\ a = \theta \end{array}\right\}$ ext. angle of cyclic ABCD
$\angle ABD = \angle CEB = z$ (common $\angle$)
$\therefore \triangle ADE \sim \triangle CBE$ *
$DA = DE$: $a = z$ (isosceles $\triangle$)
$\therefore a = z = 3\alpha$ (given $\angle BEC = 3\alpha$)

$b = a + \alpha$ [Ext. $\angle$ of $\triangle ABF$]
$\therefore b = 4\alpha$
$b + \theta + z = 180°$ [$\Sigma$ of $\angle$ in $\triangle BCE$]
$\therefore 4\alpha + 3\alpha + 3\alpha = 180° \Rightarrow \alpha = 18°$ *

6. $\frac{dv}{dt} = \sin \pi t - \sqrt{3}\cos \pi t = 2\sin\left(\pi t - \frac{\pi}{3}\right)$

$v = -\frac{2}{\pi}\cos\left(\pi t - \frac{\pi}{3}\right) + c_1$

$t = 0, v = 0$: $c_1 = \frac{1}{\pi} \Rightarrow v = -\frac{2}{\pi}\cos\left(\pi t - \frac{\pi}{3}\right) + \frac{1}{\pi}$

$x = \int v\,dt = -\frac{2}{\pi^2}\sin\left(\pi t - \frac{\pi}{3}\right) + \frac{t}{\pi} + c_2$

$t = 0, x = 0$: $x = -\frac{2}{\pi^2}\sin\left(\pi t - \frac{\pi}{3}\right) + \frac{t}{\pi} - \frac{\sqrt{3}}{\pi^2}$ *

$\frac{dv}{dt} = 0$: $\sin\left(\pi t - \frac{\pi}{3}\right) = 0$

$\therefore \pi t - \frac{\pi}{3} = 0, \pi \Rightarrow t_1 = \frac{1}{3}, t_2 = \frac{4}{3}$

$t_1 = \frac{1}{3}$: $v_1 = -\frac{2}{\pi}(1) + \frac{1}{\pi} = -\frac{1}{\pi}$ *

$t_2 = \frac{4}{3}$: $v_2 = -\frac{2}{\pi}(-1) + \frac{1}{\pi} = \frac{3}{\pi}$ *

$v = 0$: $\cos\left(\pi t - \frac{\pi}{3}\right) = \frac{1}{2} \Rightarrow t = \frac{2}{3}$

$\therefore$ Distance travelled $= \frac{1}{3\pi^2}\left[\pi + 6\sqrt{3}\right]$ *

7. $P(A \text{ hit first}) = \frac{1}{6} + \frac{5}{6}\left(\frac{4}{5}\right)\left(\frac{1}{6}\right) + \left(\frac{2}{3}\right)^2\left(\frac{1}{6}\right) + \left(\frac{2}{3}\right)^3\left(\frac{1}{6}\right) + \ldots = \frac{\frac{1}{6}}{1 - \frac{2}{3}} = \frac{1}{2}$ *

8. $P(R|A) = \frac{P(R \cap A)}{P(R \cap A) + P(R' \cap A)}$
$= \frac{\frac{1}{5}\left(\frac{1}{20}\right)}{\frac{1}{100} + \frac{4}{5}\left(\frac{1}{50}\right)} = \frac{5}{13}$ *

9. (a) $E(W) = 0.5 - 3.5 = -3$ *

$Var(W) = 1^2(0.5) + (-1)^2(3.5) = 4$ *

(b) $E(W) \neq Var(W)$: $W$ is not a Poisson random variable *

10. (a) Prob. $= {}^5C_4 (0.85)^4(0.15) = 0.3915$ *

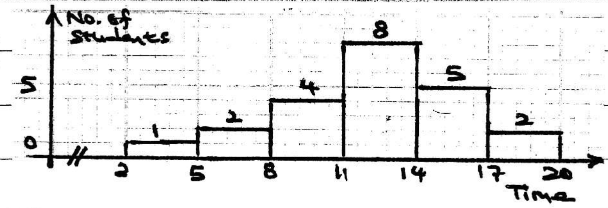
(b) $X$ = no. of patients survive

$X \sim B(200, 0.85) \rightarrow X \sim N(170, 25.5)$

$P(X > 160) = P(X > 160.5)$

$= P\left(Z > \frac{160.5 - 170}{\sqrt{25.5}}\right) = P(Z > -1.8813)$

$= 0.9700$ *

11. (a) 


Distribution: left-skewed *

(b) $\bar{x} = \frac{1}{22}[3.5 + 2(6.5) + 4(9.5) + 8(12.5) + 5(15.5) + 2(18.5)]$
$= \frac{269}{22} = 12.23$ *

$m = 11 + \left(\frac{11 - 7}{8}\right)(3) = 12.5$ *

mode $= 11 + \left(\frac{4}{4+3}\right)(3) = 12.71$ *

since $\bar{x} < m <$ mode

$\Rightarrow$ distribution is left-skewed *

12. (a) $\int_1^b \sqrt{\frac{x-1}{12}}\,dx = 1$

$\therefore \frac{1}{2\sqrt{3}}\left[\frac{(x-1)^{3/2}}{3/2}\right]_1^b = 1$

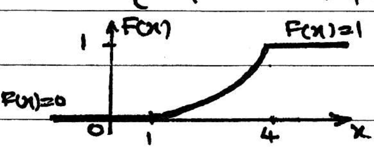
$\therefore (b-1)^{3/2} = 3\sqrt{3} \Rightarrow b - 1 = 3$

$\therefore b = 4$ *

(b) $P(X < x) = \int_1^x \frac{1}{2\sqrt{3}}(x-1)^{\frac{1}{2}}\,dx$

$= \frac{1}{3\sqrt{3}}\left[(x-1)^{3/2}\right]_1^x = \frac{1}{3\sqrt{3}}(x-1)^{3/2}$

$\therefore F(x) = \begin{cases} 0 & , x < 1 \\ \frac{1}{3\sqrt{3}}(x-1)^{3/2} & , 1 \le x \le 4 \\ 1 & , 4 < x \end{cases}$ *




(c) $E(X) = \int_1^4 \frac{x}{2\sqrt{3}}(x-1)^{\frac{1}{2}}\,dx$

$= \left[\frac{x}{2\sqrt{3}}\left(\frac{(x-1)^{3/2}}{3/2}\right)\right]_1^4 - \int_1^4 \frac{1}{2\sqrt{3}}\left(\frac{2}{3}\right)(x-1)^{3/2}\,dx$

$= \frac{1}{3\sqrt{3}}\left[4(3\sqrt{3}) - 0\right] - \frac{1}{3\sqrt{3}}\left[\frac{2}{5}(x-1)^{5/2}\right]_1^4$

$= 4 - \frac{2}{15\sqrt{3}}\left[9\sqrt{3} - 0\right] = \frac{14}{5}$ *